1

Just enough inflation

D.J.SCHWARZ* and E.RAMIREZ

Fakultät für Physik, Universität Bielefeld,
Postfach 100151, 33501 Bielefeld, Germany
E-mail: dschwarz, eramirez at physik.uni-bielefeld.de

We propose a version of chaotic inflation, in which a fundamental scale M, well below the Planck scale $M_{\rm P}$, fixes the initial value of the effective potential. If this scale happens to be the scale of grand unified theories, there are just enough e-foldings of inflation. An initial epoch of fast-roll breaks scale-invariance at the largest observable scales.

Keywords: Cosmological inflation

1. Motivation

Cosmological inflation provides an explanation for the formation of structures in the Universe. However, we lack a fundamental theory from which inflation would emerge as a natural consequence. Instead, a large number of inflationary scenarios have been proposed. On the other hand, the standard model of particle physics (SM) unifies three of the four fundamental forces of nature and requires the existence of a particle with the same properties as the field driving inflation in many models.

If the Higgs field could act as the inflaton field, current particle physics experiments at Tevatron and LHC would probe the physics of inflation. The study of this problem has first been addressed long ago.[1] It was shown that a false vacuum scenario for the Higgs field (at the electroweak energy scale) leads to an overproduction of density perturbations and is thus ruled out. An alternative would be to concentrate on much higher values of the Higgs potential in the spirit of chaotic (or large field) inflation.[2]

This scenario has been investigated in the context of a Higgs field minimally coupled to gravity, approximating the potential by its quartic term[3] and in theories with a non-minimal coupling.[4] It has been shown[5] that the SM could in principle be well defined up to the Planck scale. Then the $\lambda\phi^4$ potential, presumably the dominant contribution to the effective Higgs potential at high energies, could provide enough inflation, but generically overproduces density perturbations as $\lambda = \mathcal{O}(1)$ and is thus ruled out. Even if one allows for an arbitrary value of λ, data from 5 years of WMAP observations exclude the scenario of an extended slow-roll epoch

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2

in a $\lambda\phi^4$-potential.[6] Likewise, the inclusion of a non-minimal coupling allows the possibility of adjusting the Higgs potential so that cosmological perturbations can be in accordance with observational restrictions.[4]

2. Fundamental scales

Here we consider a different situation in which the total amount of inflation is not much more than 60 e-foldings. The onset of inflation is thus observable and violates the slow-roll assumptions. The resulting primordial power spectrum is not scale invariant, as the moment of the onset of the slow-roll regime distinguishes a scale. It turns out that such a situation cannot be discarded on grounds of current analysis and observations.[7]

This new scenario of "just enough" chaotic inflation seems to be generic if two fundamental energy scales are relevant in the very early Universe. The Planck scale M_p is the fundamental scale of quantum gravity. The notions of spatial curvature, expansion rate H and kinetic energy density are well defined and real quantities, at least up to that scale. However, this is less clear for the effective potential V of the inflaton. The effective potential carries the information about all interactions of the inflaton, except its gravitational ones. If we consider the SM Higgs as a candidate for the inflaton, it is clear that inflation would only take place if the effective Higgs potential is real — the existance of an imaginary part would lead to the immediate decay of the inflaton.

The quartic self-coupling of the Higgs runs with energy. Depending on the mass of the top-quark, the self-coupling decreases with increasing energy scale μ and can become negative at high energy (see Fig. 1). The effective Higgs potential is real as long as the quartic self-coupling is positive, but becomes complex as soon as it runs to negative values.[8] Such an imaginary contribution would give rise to the decay of the Higgs field and thus establishes an effective upper bound for an effective real Higgs potential $V < M^4$, with $M < M_P$. Motivated by the SM Higgs, we can assume that $M \sim 10^{15}$ GeV and $\lambda \ll 1$ at that scale.

3. $\lambda\phi^4$-inflation

We applied that idea to $\lambda\phi^4$-inflation.[7] As for chaotic inflation, curvature, expansion rate and kinetic energy density start at the Planck scale. However, if the potential energy were at M_P as well, the inflaton would decay and the Universe would not grow old. Only regions with $V < M^4$ (occurs with a finite probability) start to inflate. These regions are dominated by kinetic energy initially. It decays quickly and the Universe enters a state of fast-roll inflation, followed by about 60 e-foldings of slow-roll inflation. The trajectory of the Universe in the observable plane of tensor-to-scalar ratio vs. spectral tilt is shown in Fig. 2. It turns out that this model evades the constraints form the WMAP 5 year data,[6] as it breaks scale-invariance at the largest cosmological scales.[7]
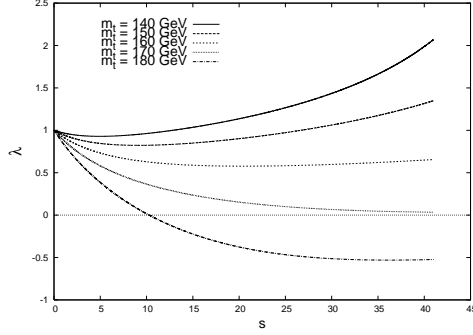
Fig. 1.   Running of λ for various choices of the (bare) top quark mass as a function of $s \equiv \ln(\mu/m_Z)$ from $s = 0$ ($\mu = m_Z$) to $s = 41$ ($\mu \approx 24M_p$). Here we assume $\lambda = 1$ and $\alpha_3 = 0.12$ at $s = 0$ and use the renormalisation group equations from the SM at one loop.

We found that by means of a fine tuned running of λ, the tree-level SM Higgs effective potential could give rise to such a scenario. However, the inclusion of one- and higher-loop corrections spoils that consistency.
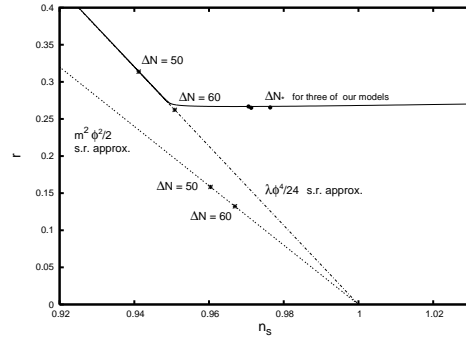

Fig. 2.   Tensor-to-scalar ratio r versus the spectral index n_{s}. The trajectory of just enough inflation violates the slow-roll approximation. Two standard slow-roll trajectories are shown for comparison.

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4

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